Why to study Linear Algebra?

What is Linear Algebra ?

- Study of Vectors and Matrices, combinations of Vectors and Matrices, and linear transformations between them
- Can also be viewed as study of 1D lines, 2D planes, 3D space and similar hyper-geometries in higher dimensions
- In the modern, data-driven world, Linear Algebra is also referred to as the ‘Mathematics of Data’
- In very simple terms, it can be called as the branch of mathematics that deals with finding (approximate) solutions to a system of ‘*m*’ linear equations in ‘*n*’ variables of the form $Ax = b$
- The freedom in choosing *‘m’*, *‘n’*, *‘A’* and *‘b’* to define a system, leads us to a vast majority of concepts and algorithms in LA to solve the above system

LA → Linear Algebra

Why Linear Algebra ?

- Like with any other form of mathematics, LA too is a fundamental tool for many form of sciences
- The power of Linear Algebra is extensively exploited by the core concepts in engineering, be it any branch
- It is a computer friendly form of mathematics
- The discrete representation of systems in matrix-vector forms is compatible with the discrete, digital architecture of electronic devices
- Ever heard of Artificial Intelligence or Machine Learning, that famous AI/ML abbreviation every engineer fantasizes these days? It stands on a robust foundation of Linear Algebra !!
- Many programming languages have dedicated tools for performing linear algebraic operations on data - Numerical Linear Algebra

Applications of Linear Algebra

- Basic systems we have studied in high school, which we can solve elegantly using LA
  - Free Body Diagrams
  - KCL and KVL in Electrical Circuits
  - Representation and manipulation of geometric entities like lines, planes, 3D space
  - Representing conic sections like circle, ellipse, hyperbola, etc
- Optimization of production and resource utilization in industries - Linear Programming
- Animations - possible only due to linear algebra
  - Scaling and Rotation of images and animated objects
  - Change in perspective of viewing an object
  - Augmented Reality and Virtual Reality
- Control Systems Modeling
  - Stability Analysis of a System
  - Fibonacci Series - Calculating the Golden Ratio

Applications of Linear Algebra

- Signal Processing - Fourier Analysis (Linear Algebra for functions instead of numbers)
- Probability and Statistics
  - Use of Vector-Matrix representation in multivariate statistics
  - Multi-dimensional Gaussian distributions
  - Study of stochastic (read probabilistic) systems
    - Population growth and migration
    - Pandemic growth rate prediction !!
    - Economic Growth, stock market prediction, etc
- Graphical Network Analysis - Google PageRank Algorithm and Eigen Vectors
- Pure Sciences
  - Solving Differential Equations using Numerical Techniques
  - Functional Analysis - Wave Functions in Quantum Mechanics

Applications of Linear Algebra

- Robotics
  - Robotic Manipulation
    - 3D Rotations
    - Homogeneous transformations
    - Study of manipulator kinematics and dynamics
  - Computer Vision
    - Domain Transforms on Images
    - Image Feature Extraction
    - 3D reconstruction
  - Mobile Robots
    - Point cloud data from Laser Scans and Depth Cameras
    - Path Planning

Applications of Linear Algebra

- Machine Learning and Artificial Intelligence
  - Regression - Solving the $Ax = b$ problem, again
  - Neural Networks - feed forward and backpropogation
  - Dimensionality Reduction of data - SVD, PCA
  - Optimization of desired Loss functions for training models

There are many more such domains and applications in engineering and sciences where knowledge of linear algebra is assumed.

Why should you believe us ?

Right, do not believe us blindly, but see it for yourself the prerequisites of some famous courses and programs offered by famous universities

- CS229: Machine Learning by Stanford University
- CS230: Deep Learning by Stanford University
- EE364A: Convex Optimization by Stanford University
- Carnegie Mellon University (CMU) - MRSD program
- 16-385 Computer Vision by CMU
- 16-833 Robot Localization and Mapping by CMU
- ECE6550 Linear Systems and Controls by Georgia Tech
- CS419 Introduction to Machine Learning by IITB

Who will teach you ?

- VJTI is all about self-study
- Very limited branches exclusively teach LA as an independent course here in VJTI, that too in your last semester only as an elective
- But do not worry, we have Prof. Gilbert Strang of the MIT available on youtube for free
- There ain't no better person alive to learn LA from than Dr. Strang
- Also, for an intuitive perspective of Linear Algebra, do search for the 'Essence of Linear Algebra' playlist on the youtube channel '**3Blue1Brown**' by 'Grant Sanderson'

Some cool things you will understand from Dr. Strang's lectures

1. Derivation of the formula $|A - \lambda I| = 0$ which is used to calculate Eigen Values of a square Matrix 'A'
    - Relies on the concept of Vector Subspaces and matrix ranks
2. Linear Regression - Least squares approximation and allied topics in Machine Learning
    - Relies on the concept of Vector Subspaces and projections of vectors onto these subspaces
3. Determine the stability of a linear system, model a Fibonacci Series, calculate the Golden Ratio, and solve a differential equation
    - Uses Eigen decomposition of matrices
4. Representation of conics sections, finding extrema of second order curves using matrices
    - Relies on concept of Positive Definite matrices
5. Dimensionality reduction and Principal Component Analysis in Data Science
    - Based on Singular Value Decomposition

What is our role at SRA and What is expected of you ?

- Our Role
  - Provide you with all the materials required to study the subject
  - Point you towards the right online resources
  - Provide you with assignments and tutorials
  - Try to solve your doubts to the best of our ability

- Your Role
  - Study
  - Discuss
  - Solve
  - Ask doubts

Thank you